

22. No. In order for this equality to be true, A , B , and C would have to be collinear.

Exploration (pages 32–33)

- The points are the same. Each represents a location on the surface.
- A geodesic in Euclidean geometry extends infinitely in each direction. The geodesic on the sphere is a circle.
- Two intersecting geodesics in Euclidean geometry form four angles. Two intersecting

geodesics on a sphere form eight angles. The sides of these angles do not extend infinitely.

4. The sum of the measures of the angles of a triangle in Euclidean geometry is 180° . Drawing three geodesics on a sphere forms eight triangles. The sum of the measures of the angles of a triangle on sphere is greater than 180° .

Chapter 2. Logic

2-1 Sentences, Statements, and Truth Values (pages 39–41)

Writing About Mathematics

- A sentence in grammar need only have a subject and a predicate. A mathematical sentence used in logic must state a fact or a complete idea that can be judged to be true or false.
- Answers will vary.
 - Example: Today is Wednesday.
 - Example: I am six feet tall.
 - Example: This country is in the southern hemisphere.

Developing Skills

- Mathematical sentence
- Mathematical sentence
- Not a mathematical sentence
- Mathematical sentence
- Not a mathematical sentence
11. She 12. We 13. y
14. x 15. This 16. He
17. It 18. It 19. a. True
20. a. Open 21. a. False 22. a. True
b. They
23. a. Open 24. a. False 25. a. True
b. x
26. a. False 27. {New York}
28. {Nevada, Illinois}
29. {Massachusetts, New York}
30. {Nevada, Illinois, Massachusetts, Alaska, New York}
31. {Alaska} 32. {triangle} 33. \emptyset
34. {triangle} 35. {square, rectangle}
36. {square, rhombus}
37. {square, rectangle, parallelogram, rhombus}
38. {trapezoid}

- {square, rectangle, parallelogram, rhombus, trapezoid}
- The school does not have an auditorium.
- A stop sign is not painted red.
- The measure of an obtuse angle is not greater than 90° .
- There are not 1,760 yards in a mile.
- Michigan is a city. 45. $14 \times 2 - 16 \neq 12$
- $3 + 4 + 5 = 6$ 47. Today is Wednesday.
48. a. p 49. a. $\sim p$ 50. a. q
b. True b. False b. False
51. a. $\sim q$ 52. a. r 53. a. $\sim r$
b. True b. Open b. Open
54. a. $\sim q$ 55. a. p 56. a. q
b. True b. True b. False
57. a. Summer does not follow spring. b. False
58. a. August is not a summer month. b. False
59. a. A year does not have 12 months. b. False
60. a. She does not like spring. b. Open
61. a. Summer follows spring. b. True
62. a. August is a summer month. b. True
63. a. A year has 12 months. b. True
64. a. She likes spring. b. Open

2-2 Conjunctions (pages 46–48)

Writing About Mathematics

1. No. As shown in the following truth table, $\sim(p \wedge q)$ and $\sim p \wedge \sim q$ have different truth values.

p	q	$(p \wedge q)$	$\sim(p \wedge q)$	$\sim p$	$\sim q$	$\sim p \wedge \sim q$
T	T	T	F	F	F	F
T	F	F	T	F	T	F
F	T	F	T	T	F	F
F	F	F	T	T	T	T

2. p , q , and r must all be true because, for a conjunction to be true, each of the conjuncts must be true.

Developing Skills

3. $p \wedge q$ 4. $p \wedge r$ 5. $\sim p$
 6. $\sim p \wedge r$ 7. $q \wedge \sim r$ 8. $\sim p \wedge \sim q$
 9. $\sim r \wedge \sim p$ 10. $\sim r \wedge p$ 11. $\sim(p \wedge q)$
 12. $\sim(q \wedge \sim p)$ 13. False 14. False
 15. True 16. True 17. False
 18. False 19. True 20. False
 21. True 22. False 23. False
 24. True, True 25. True, False 26. False, True
 27. True 28. True

Applying Skills

29. True 30. False 31. Uncertain
 32. False 33. True 34. False
 35. Uncertain 36. False
 37. a. True 38. a. True
 b. True b. True
 c. False c. False

2-3 Disjunctions (pages 51–53)

Writing About Mathematics

- The truth set of the negation of a statement contains the elements of the replacement set that make the statement false. The complement of a set contains the elements of the universe that are not members of the set.
- When two statements are connected by the *inclusive or*, the disjunction is true when one of the statements is true or both of the statements are true. When two statements are connected by the *exclusive or*, the disjunction is true only in the case that exactly one of the statements is true.

Developing Skills

3. a. $m \vee k$ 4. a. $c \vee l$ 5. a. $c \vee m$
 b. True b. True b. True
 6. a. $\sim k \vee \sim c$ 7. a. $l \vee k$ 8. a. $l \wedge c$
 b. False b. True b. False
 9. a. $\sim(c \vee m)$ 10. a. $\sim(\sim k \vee l)$
 b. False b. True
 11. a. $c \wedge k$ 12. a. $(\sim c \vee \sim m) \wedge l$
 b. True b. False
 13. a. Spring is a season or Halloween is a season.
 b. True
 14. a. Breakfast is a meal and spring is a season.
 b. True
 15. a. Spring is not a season or Halloween is a season.
 b. False
 16. a. Breakfast is a meal and Halloween is not a season.
 b. True
 17. a. Breakfast is not a meal or spring is not a season.
 b. False

18. a. It is not the case that spring is a season and Halloween is a season.
 b. True
 19. a. It is not the case that breakfast is a meal or spring is not a season.
 b. False
 20. a. Breakfast is not a meal and spring is not a season.
 b. False
 21. True 22. True 23. False
 24. False, True 25. True, False 26. False
 27. True

Applying Skills

28. True 29. True 30. False
 31. Uncertain 32. True

2-4 Conditionals (pages 58–60)

Writing About Mathematics

- Let p be 8 is divisible by 4.
 (True: $8 \div 4 = 2$)
 Let q be 8 is divisible by 2.
 (True: $8 \div 2 = 4$)
 $p \rightarrow q$ (T \rightarrow T) is true.
 - Let p be 6 is divisible by 4.
 (False: $6 \div 4 = 1.5$)
 Let q be 6 is divisible by 2.
 (True: $6 \div 2 = 3$)
 $p \rightarrow q$ (F \rightarrow T) is true.
 - Let p be 7 is divisible by 4.
 (False: $7 \div 4 = 1.75$)
 Let q be 7 is divisible by 2.
 (False: $7 \div 2 = 3.5$)
 $p \rightarrow q$ (F \rightarrow F) is true.
- No. Every number that is divisible by 4 is also divisible by 2 because the prime factorization of 4 is 2×2 .
- The truth value is the same when p and q are both true or when p and q are both false.

p	q	$p \rightarrow q$	$q \rightarrow p$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

Developing Skills

3. a. A polygon is a square.
 b. A polygon has four right angles.
 4. a. It is noon.
 b. It is time for lunch.

5. a. You want help.
b. You ask a friend for help.
6. a. You are not interrupted.
b. You will finish more quickly.
7. a. The length of one side of a square is s .
b. The perimeter of a square is $4s$.
8. a. Many people work at a task.
b. The task will be completed quickly.
9. a. $2x + 7 = 11$
b. $x = 2$
10. a. You do not get enough sleep.
b. You will not be alert.
11. $p \rightarrow r$ 12. $q \rightarrow r$ 13. $\sim p \rightarrow \sim r$
14. $\sim q \rightarrow \sim r$ 15. $q \rightarrow p$ 16. $p \rightarrow r$
17. a. $t \rightarrow l$ 18. a. $l \rightarrow t$ 19. a. $l \rightarrow b$
b. True b. False b. True
20. a. $\sim b \rightarrow l$ 21. a. $\sim t \rightarrow \sim l$ 22. a. $b \rightarrow t$
b. True b. False b. False
23. a. $(\sim b \wedge t) \rightarrow l$ 24. a. $(b \wedge \sim t) \rightarrow \sim l$
b. True b. False
25. True 26. True 27. True
28. True 29. True 30. True
31. False
32. a. If July is a warm month, then I work in my garden.
b. True
33. a. If I am busy every day, then I do not work in my garden.
b. True
34. a. If I like flowers, then I work in my garden.
b. True
35. a. If I do not work in my garden, then July is not a warm month.
b. True
36. a. If July is a warm month and I like flowers, then I am busy every day.
b. False
37. a. If July is a warm month and I work in my garden, then I like flowers.
b. True
38. a. If July is not a warm month, then I am busy every day and I like flowers.
b. True
39. a. If I work in my garden, then July is a warm month or I am not busy every day.
b. True
40. $p \rightarrow q$ 41. $q \rightarrow p$ 42. True, False
43. True 44. True 45. True

Applying Skills

46. True 47. False 48. Uncertain
49. Uncertain 50. True

2-5 Inverses, Converses, and Contrapositives (pages 67–69)

Writing About Mathematics

- No. A conditional ($p \rightarrow q$) can be true when p is false and q is true. However, in the converse ($q \rightarrow p$) of this case, $F \rightarrow T$ is false.
- Yes. A conditional and its contrapositive are logical equivalents, so they have the same truth values. Also, the converse of a statement and the inverse of a statement are logical equivalents, so they have the same truth values.

Developing Skills

- a. $\sim p \rightarrow \sim q$ 4. a. $\sim t \rightarrow w$
b. $q \rightarrow p$ b. $\sim w \rightarrow t$
c. $\sim q \rightarrow \sim p$ c. $w \rightarrow \sim t$
- a. $m \rightarrow \sim p$ 6. a. $p \rightarrow q$
b. $p \rightarrow \sim m$ b. $\sim q \rightarrow \sim p$
c. $\sim p \rightarrow m$ c. $q \rightarrow p$
- a. If 6 is not greater than 3, then -6 is not greater than -3 .
b. False c. True
- a. If a trapezoid is not a parallelogram, then a trapezoid does not have exactly two pairs of parallel sides.
b. True c. True
- a. If $3(3) \neq 9$, then $3(4) \neq 12$.
b. True c. True
- a. If $2^2 \neq 4$, then $3^2 \neq 6$.
b. False c. True
- If you eat Quirky oatmeal, then you lower your cholesterol.
- If you get rich, then you enter the Grand Prize drawing.
- If your hair curls, then you use Shiny's hair cream.
- If your pet grows three inches, then you feed him Crazy Kibble.
- a. If a number is exactly divisible by 2, then the number is even.
b. True c. True
- a. If 0.75 is rational, then it is an integer.
b. True c. False
- a. If $8^2 = 1^2 + 7^2$, then $8 = 1 + 7$.
b. False c. True
- a. If $4(5) - 6 = 14$, then $4(5) - 6 = 20 - 6$.
b. True c. True
- a. If Rochester is not the capital of New York, then Rochester is not a city.
b. False c. False
- a. If two angles are not supplementary, then they do not form a linear pair.
b. True c. True

21. a. If $4 - 3 \neq 2$, then $3 - 2 \neq 1$.
 b. False c. False
22. a. If a triangle is not equiangular, then all angles of the triangle are not equal in measure.
 b. True c. True
23. a. If $\frac{1}{2}$ is not a counting number, then $\frac{1}{2}$ is not greater than 0.
 b. False c. False
24. (4) 25. (3) 26. (2) 27. (3) 28. (2)

Applying Skills

29. a. If Derek lives in Nevada, then he lives in Las Vegas; sometimes true.
 b. If Derek does not live in Las Vegas, then he does not live in Nevada; sometimes true.
 c. If Derek does not live in Nevada, then he does not live in Las Vegas; always true.
30. a. If the probability of picking a red marble from a bin is $\frac{1}{2}$, then the bin contains 3 red marbles and 3 blue marbles; sometimes true.
 b. If a bin does not contain 3 red marbles and 3 blue marbles, then the probability of picking a red marble from the bin is not $\frac{1}{2}$; sometimes true.
 c. If the probability of picking a red marble from a bin is not $\frac{1}{2}$, then the bin does not contain 3 red marbles and 3 blue marbles; always true.
31. a. If a polygon is an octagon, then it has eight sides; always true.
 b. If a polygon does not have eight sides, then it is not an octagon; always true.
 c. If a polygon is not an octagon, then it does not have eight sides; always true.
32. a. If a garden grows vegetables, then it grows carrots; sometimes true.
 b. If a garden does not grow carrots, then it does not grow vegetables; sometimes true.
 c. If a garden does not grow vegetables, then it does not grow carrots; always true.
33. a. If the area of a rectangle is 48 square feet, then the dimensions of the rectangle are 8 feet by 6 feet; sometimes true.
 b. If the dimensions of a rectangle are not 8 feet by 6 feet, then the area of the rectangle is not 48 square feet; sometimes true.
 c. If the area of a rectangle is not 48 square feet, then the dimensions of the rectangle are not 8 feet by 6 feet; always true.
34. a. If a number is divisible by 7, then it has 7 as a factor; always true.
 b. If a number does not have 7 as a factor, then it is not divisible by 7; always true.

- c. If a number is not divisible by 7, then it does not have 7 as a factor; always true.

2-6 Biconditionals (pages 73–74)

Writing About Mathematics

- A prime number is a whole number greater than 1 if and only if the prime number has exactly two factors.
- Yes. A biconditional is false when the truth value of p is not the same as the truth value of q . When this is the case, the conditional $p \rightarrow q$ has a different truth value from its converse.

Developing Skills

- | | | |
|-----------|----------|-----------|
| 3. True | 4. False | 5. True |
| 6. True | 7. True | 8. True |
| 9. True | 10. True | 11. False |
| 12. False | 13. True | 14. False |
| 15. False | 16. True | |

Applying Skills

17. a. x is divisible by 2 and 3 if and only if x is divisible by 6.
 b. Let $x = 2$; 2 is divisible by 2 but not by 3 or 6;
 $(p \wedge q) \leftrightarrow r$ is $F \leftrightarrow F$, which is true.
 Let $x = 3$; 3 is divisible by 3 but not by 2 or 6;
 $(p \wedge q) \leftrightarrow r$ is $F \leftrightarrow F$, which is true.
 Let $x = 5$; 5 is not divisible by 2, 3, or 6;
 $(p \wedge q) \leftrightarrow r$ is $F \leftrightarrow F$, which is true.
 Let $x = 6$; 6 is divisible by 2, 3, and 6;
 $(p \wedge q) \leftrightarrow r$ is $T \leftrightarrow T$, which is true.
 Let $x = 8$; 8 is divisible by 2 but not by 3 or 6;
 $(p \wedge q) \leftrightarrow r$ is $F \leftrightarrow F$, which is true.
 Let $x = 9$; 9 is divisible by 3 but not by 2 or 6;
 $(p \wedge q) \leftrightarrow r$ is $F \leftrightarrow F$, which is true.
 Let $x = 11$; 11 is not divisible by 2, 3, or 6;
 $(p \wedge q) \leftrightarrow r$ is $F \leftrightarrow F$, which is true.
 Let $x = 12$; 12 is divisible by 2, 3, and 6;
 $(p \wedge q) \leftrightarrow r$ is $T \leftrightarrow T$, which is true.
- c. Yes. The prime factorization of 6 is 2×3 . If a counting number is divisible by both of these factors, then it is divisible by 6. If a counting number is not divisible by both of these factors, then it is not divisible by 6.
18. a. Yes b. Yes c. Yes d. Yes e. No
19. A triangle is isosceles if and only if it has two congruent sides; true.
20. Two angles are both right angles if and only if they are congruent; false.
21. Today is Thursday if and only if tomorrow is not Saturday; false.
22. Today is not Friday if and only if tomorrow is not Saturday; true.

2-7 The Laws of Logic (pages 79–80)

Writing About Mathematics

1. Yes. If $p \rightarrow q$ is false, then q must be false. If $q \vee r$ is true, and q is false, then by the Law of Disjunctive Inference, r must be true.
2. Yes. When $\sim q$ is true, then q is false. If $p \vee q$ is true, and q is false, then by the Law of Disjunctive Inference, p must be true. Since p is true and $\sim q$ is true, $p \wedge \sim q$ is also true.

Developing Skills

3. True by the Law of Disjunctive Inference
4. True by the Law of Detachment
5. Cannot be found to be true or false because one disjunct is known to be true
6. True by the Law of Disjunctive Inference
7. True by the Law of Detachment
8. Cannot be found to be true or false because the hypothesis is false
9. True by the Law of Detachment
10. Cannot be found to be true or false because the hypothesis is false
11. False; the truth values of both statements in a true biconditional must be the same.
12. Cannot be found to be true or false because the conclusion is true
13. True by applying the Law of Detachment to the contrapositive of the conditional
14. Cannot be found to be true or false because the hypothesis is false

Applying Skills

15. I take band; the Law of Detachment.
16. $\sqrt{6}$ is irrational; the Law of Disjunctive Inference.
17. $b = 4$; the Law of Detachment.
18. It is not 8:15 A.M.; the Law of Detachment can be applied to the contrapositive of the conditional.
19. No conclusion; a conditional with a true conclusion may have either a true hypothesis or a false hypothesis.
20. x is even and a prime; a biconditional with a true conclusion has a true hypothesis.
21. It is February, and it is not summer; in a true conjunction, both statements must be true. The Law of Disjunctive Inference may be applied to the disjunction.
22. No conclusion; a conditional with a true conclusion may have either a true hypothesis or a false hypothesis.
23. Last Saturday we flew kites; the Law of Disjunctive Inference.
24. I study computer science and I take welding; the

Law of Disjunctive Inference.

25. Five has exactly two factors; a biconditional with a true hypothesis has a true conclusion.
26. x is not an integer greater than 2 and prime; in a true conjunction, both statements must be true. The Law of Detachment can be applied to the contrapositive of the conditional.
27. Ray DF does not bisect angle CDE ; the hypothesis of the contrapositive is true, so its conclusion must be true, that is, the hypothesis of the conditional must be false.

2-8 Drawing Conclusions (pages 83–85)

Writing About Mathematics

1. Yes. Both members of the conjunction must be true. Since p is false, q is true by the Law of Disjunctive Inference. Also, r is true.
2. No. Both members of the conjunction must be true. Since the conjunct r is false, the truth values of p and q cannot be determined.

Developing Skills

3. True
4. True
5. True
6. p is true and q is true.
7. p is true and q is false.
8. p is false, and q cannot be determined.
9. True

Applying Skills

10. Laura is an investment manager. Marta is a doctor. Shanti is a lawyer.
11. Alex is a plumber. Tony is a bookkeeper. Kevin is a teacher.
12. Gamma
13. Ren had chicken pot pie. Logan had pizza. Kadoogan had a ham sandwich.
14. Zach plays baseball. Steve plays soccer. David plays basketball.
15. Taylor studies Latin. Melissa studies French. Lauren studies Spanish.
16. Augustus is a truth teller. Brutus is a liar. Caesar is a liar.

Review Exercises (pages 87–89)

In 1 and 2, answers will vary. Examples are given.

1. I go to school. I play basketball. If I go to school, then I play basketball. If I play basketball, then I go to school. If I do not play basketball, then I go to school.
2. I do not play basketball. I do not go to school. If I do not go to school, then I play basketball. I go to school and I do not play basketball. I do not go to school and I play basketball.

3. No. The biconditional $p \leftrightarrow q$ is true when $p \rightarrow q$ and $q \rightarrow p$ are both false or both true. The contrapositives $\sim q \rightarrow \sim p$ and $\sim p \rightarrow \sim q$ have the same truth values as the conditionals because they are logically equivalent. Therefore, $\sim p \leftrightarrow \sim q$ has the same truth values as $p \leftrightarrow q$.
4. a. At first you don't succeed.
b. You should try again.
5. a. You are late one more time.
b. You will get a detention.
6. True 7. False 8. True
9. False 10. True 11. False
12. True 13. 5 14. p
15. True 16. False, True 17. True
18. False 19. False 20. False
21. True 22. True
23. {6, 7, 8, 9, 10} 24. {1, 2, 3, 4, 5}
25. {2, 3, 5, 7} 26. {1, 4, 6, 8, 9, 10}
27. {2, 3, 5, 6, 7, 8, 9, 10} 28. {7}
29. {2, 3, 5} 30. {1, 2, 3, 4, 5, 6, 8, 9, 10}
31. {1, 2, 3, 4, 5, 7} 32. {7}
33. a. If I do not live in Oregon, then I do not live in the Northwest.
b. If I live in the Northwest, then I live in Oregon.
c. If I do not live in the Northwest, then I do not live in Oregon.
d. I live in Oregon if and only if I live in the Northwest.
34. $\angle A$ is not the vertex angle of isosceles $\triangle ABC$.
35. Janice, Sarah, Laurie
36. Judy, Janice, Sue, Sarah, Laurie
37. 1st Virginia, 2nd Kay, 3rd Janice
38. Peter plays the violin and tennis. Carlos plays the cello and soccer. Ralph plays the flute and baseball.
39. Answers will vary. Examples are given.
a. (1) 6 (2) Not possible
 (3) 4 (4) 5
b. Since there does not exist a number that is divisible by 6 but not divisible by 2, $p \rightarrow q$ is always true.
40. Peter

Exploration (page 90)

1. a. 1 and 3 2. a. 2 and 3
b. Answers will vary. b. Answers will vary.
3. a. Answers will vary. b. Answers will vary.
c. Because the hypothesis is always false, the conditional is true whether the conclusion is true or false.

Cumulative Review (pages 90–92)

Part I

1. 3 2. 2 3. 3 4. 3 5. 1
6. 3 7. 4 8. 4 9. 3 10. 1

Part II

11. "I win the ring toss game" must be true and "If I win the ring toss game, then I get a goldfish" must be true. The Law of Detachment states that if the conditional is true and the hypothesis is true, then the conclusion is true.
12. 3 ; RS is half the length of RT . Since $RS = |-5 - (-1)| = 4$, T is located 4 units from S . T is located at $-1 + 4 = 3$.

Part III

13. \overleftrightarrow{AB} refers to line AB . \overrightarrow{AB} refers to the ray that has endpoint A and contains point B . \overline{AB} refers to the line segment that has endpoints A and B . AB refers to the length of the segment \overline{AB} .
14. By definition, the measure of straight angle ABC is 180° . \overrightarrow{BD} bisects $\angle ABC$, so it splits the angle into two congruent parts, each of which measures $180 \div 2 = 90^\circ$. Since \overrightarrow{BD} intersects $\angle ABC$ to form 90° , or right, angles, we can say that \overrightarrow{BD} is perpendicular to $\angle ABC$.

Part IV

15. $a(2b + 1) = a(2b) + a(1)$ Distributive property
 $= (a \cdot 2) \cdot b + a(1)$ Associate property
 $= (2 \cdot a) \cdot b + a(1)$ Commutative property
 $= 2ab + a$ Identity property
16. a. If x is divisible by 4, then x is divisible by 12.
b. If x is not divisible by 12, then x is not divisible by 4.
c. The converse can be false: 8 is divisible by 4 but not by 12.
The inverse is always true. Since 4 is a factor of 12, every number that is divisible by 12 is also divisible by 4.
d. If x is not divisible by 4, then x is not divisible by 12.