Natural Logs & Solving Log Equations

log naturale

Logarithms with base e are called **natural logarithms.**
Natural logarithms are denoted by ln.

$\ln x = \log_e x$

1) Find ln (37) = 3.6109179126442 $e^x = 37$

2) Find N if $N = \ln(e^2)$

$e^N = e^2$

$N = 2$

3) Find ln (1) = x

$e^x = 1$

$e^x = e^0$

$\boxed{x = 0}$ ☺

$e^? = 1$

$e^0 = 1$ ✓

4) Find ln (e) = y

$e^y = e^1$

$\boxed{y = 1}$

Remember: $\ln 1 = 0$ and $\ln e = 1$

$\ln(ab) = \ln a + \ln b$

$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

$\ln a^b = b \ln a$

$\ln 1 = 0$

$\ln e = 1$

$\ln e^x = x$

$\log\left(\frac{6}{x}\right)$

$\frac{6}{x}$

Express as a single logarithm:

$\frac{1}{2}[(4\ln a + \ln b) - 4\ln c] = [4\ln a + \ln b) - 4\ln c]^{\frac{1}{2}}$

$\ln a^4 + \ln b - \ln c^4$

$\ln\left(\frac{a^4 b}{c^4}\right)^{\frac{1}{2}}$

Solving Log Equations:

A logarithmic equation can be solved using the properties of logarithms along with the use of a common base.

Properties of Logs:	
$\log_b(m \cdot n) = \log_b m + \log_b n$	$\ln(ab) = \ln a + \ln b$
$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$	$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$
$\log_b m^r = r \log_b m$	$\ln a^b = b \ln a$

To solve most logarithmic equations:
1. Isolate the logarithmic expression. (you may need to use the properties to create one logarithmic term)
2. Rewrite in exponential form (with a common base)
3. Solve for the variable.

Things to remember about logs:

$\log_b 1 = 0$	$\ln 1 = 0$
$\log_b b = 1$	$\ln e = 1$
$\log_b b^x = x$	$\ln e^x = x$

Solve for x:

1)
$$\frac{3\log(x+4)}{3} = \frac{6}{3}$$
$$\log_{10}(x+4) = 2$$

$$10^2 = x+4$$
$$100 = x+4$$
$$-4 \quad -4$$
$$96 = x$$

$$\log_2 8 = 3$$
$$2^3 = 8$$

2)
$$\ln x = 4$$
$$e^4 = x$$
$$= x$$

54.598150033144

3)
$$\log_5(x+1) = 2$$
$$5^2 = x+1$$
$$25 = x+1$$
$$-1 \quad -1$$
$$24 = x$$

4)
$\frac{2\ln(3x)}{2} = \frac{18}{2}$

$\ln_e(3x) = 9$

$\frac{e^9}{3} = \frac{3x}{3}$

$x =$ 2701.0279758585

5)
$\log_9 x + \log_9(x-8) = 1$

$\log_9 x(x-8) = 1$

$\log_9(x^2-8x) = 1$

$9^1 = x^2 - 8x$

$-9 \qquad -9$

$0 = x^2 - 8x - 9$

$0 = (x+1)(x-9)$

$\cancel{x = -1}$ | $\boxed{x = 9}$ ✓

$-1 \cdot 9$
$\boxed{+1 \cdot 9}$
$-3 \cdot 3$
$+3 \cdot 3$

6) $\ln(2x-3) + \ln(x+4) = \ln(2x^2+11)$

$\ln\left((2x-3)(x+4)\right) = \ln(2x^2+11)$

$(2x-3)(x+4) = 2x^2+11$

$\cancel{2x^2} + 8x - 3x - 12 = \cancel{2x^2} + 11$

$-2x^2 \qquad -2x^2$

$5x - 12 = 11$